Thermodynamics and Statistical Physics

Part I – Thermodynamics

Resit

Wednesday, April 13 2022, 8:30-10:30, Aletta Jacobshal

The total number of points that can be reached in this exam is 90.

Final grade = (points/10) + 1.

PROBLEM 1

Score: a+b+c+d+e.. =4+4+6+6+10=30

a) Describe the third law of thermodynamics in your own words. **(4 pt)**
b) Explain why reversible heat engines have the highest efficiency and why efficiency is always smaller than 100%. **(4 pt)**
c) What is a meant by the term "reversible process" in thermodynamics and why is it relevant? **(6 pt)**
d) Consider a gas of molecular hydrogen (H_2). We can assume it behaves as an ideal gas. Explain, under which conditions the molar heat capacity equals to $C_{V,m}=5/2R$. **(6 pt)** $\frac{5}{2}R$
e) A sample consisting of 2 mol H_2 is expanded isothermally at 0 °C from 8 dm^3 to 27 dm^3 i) reversibly, ii) against a constant external pressure (which is equal to the final pressure of the gas) and iii) freely, against zero external pressure. For all processes, calculate ΔU, w and q. You can assume H_2 to behave as an ideal gas. (Hint: How does the internal energy of an ideal gas depend on volume and pressure?) **(10 pt)**

PROBLEM 2

Score: a+b+c= 12+12+6=30

a) The number of gas molecules per unit volume that have speeds between v and dv and travel under angles between θ and $d\theta$ to a chosen direction is given by $nf(v)dv\frac{1}{2}sin\theta d\theta$. $f(v)$ is the speed distribution of the molecules. Use this information to determine the number of molecules hitting a unit area of wall in unit time (use a sketch!). **(12 pt)**
b) Use kinetic theory and the result from a) to define the coefficient of viscosity of a gas. Show that this coefficient of viscosity has the following proportionality:

$$\eta \propto nm\lambda\langle v\rangle$$

where n is particle density of the gas, m is the particle mass, λ is the mean free path of the gas molecules and $\langle v\rangle$ is their mean speed. Use a sketch! **(12 pt)**

c) It is experimentally found, that over a wide pressure range, viscosity is independent of pressure. Below which approximate pressure will viscosity become pressure dependent when measured with a device from 1660, e.g. a pendulum in an air-filled vessel (for air at 293 K, η = 18.2 μN s m^{-2})? Hint:

The dimensions of the experiment are key, here. You may assume a realistic order of magnitude of the pendulum size. (**6 pt**)

PROBLEM 3

Score: a+b+c+d+... =10+10+10 =30

Consider 1 mol of ideal gas in a state A with volume $V_A = V_0$, pressure $p_A = p_0$ and temperature $T_A = T_0 = 300\text{K}$. Consider the following thermodynamic cycle:

$$A \rightarrow B \rightarrow C \rightarrow A$$

Step 1: reversible adiabatic expansion from *A* to *B*.

Stap 2: reversible compression at constant volume from *B* to *C*.

Stap 3: reversible compression at constant pressure from *C* to *A*.

In state *B*, the gas has a pressure p_B and a volume $V_B = 2V_0$. In state *C*, the gas has a pressure $p_C = p_0$ and a volume $V_C = 2V_0$. The heat capacities are given by $C_{p,m} = \frac{7}{2}R$ and $C_{p,m} - C_{V,m} = R$.

~~a)~~ Sketch this thermodynamic cycle in a $p - V$ diagram. Indicate in which steps heat flows and in which direction (into the system and out of the system). (**10 pt**)

b) Show that for the reversible adiabatic expansion of an ideal gas, initial and final temperature are related by $T_f = T_i\left(\frac{V_i}{V_f}\right)^{1/c}$ with $c = C_{V,m}/R$. (**10 pt**)

~~c)~~ Show that the temperatures of the system in states *B* and *C are* $T_B = 227.4$ K and $T_C = 600\text{K}$. (**10 pt**)

$600 = 227\left(\frac{1}{4}\right)^{7/5}$

$\left(\frac{2C}{4}\right) = \left(\frac{600}{227}\right)^{1/1}$

Constants:
Avogardro's number: N_A=6.02 10^{23} mol^{-1}
Boltzmann constant: k_B=1.381 10^{-23} J\K
Gas constant: R=8.31 $JK^{-1}mol^{-1}$
Atomic mass unit (u): m_u=1.67 10^{-27} kg
Electronvolt: 1 eV=1.6 10^{-19} J